Unique Paper Code : 32347508

Name of the Paper : Combinatorial Optimization

Name of the Course : B.Sc (H) Computer Science

Semester : V Sem

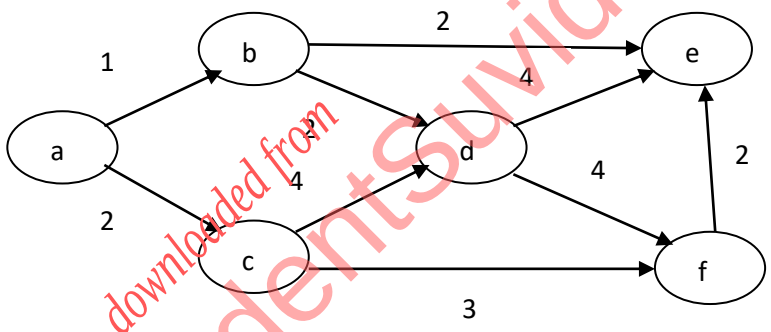
Duration : 3 Hours

Maximum Marks : 75 Marks

Instructions for Candidates:

- All questions carry equal marks.
- Attempt any **four**.

1. Formulate a linear programming problem to maximize the flow of data from node 1 to node 6 for the following network.



List all basic solutions for the following linear programming problem. Determine the feasible solutions amongst them. Also find the optimal solution.

Maximize $z = 3x_1 + 4x_2 + 10x_3$

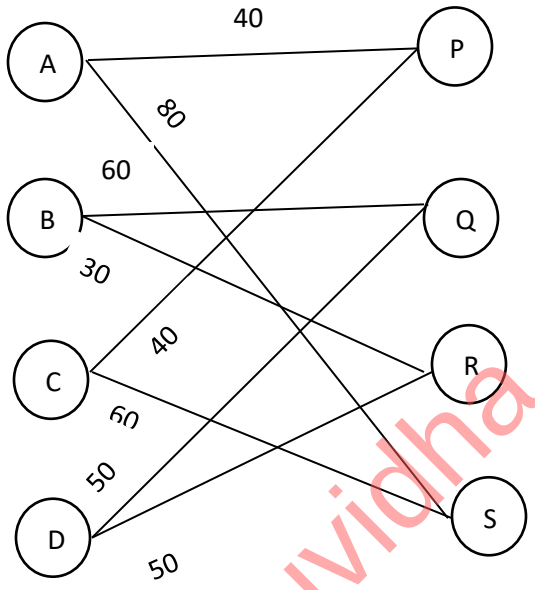
subject to,

$x_1 + x_2 + 3x_3 \leq 15$

$2x_1 + 3x_2 + x_3 \leq 45$

$x_1, x_2, x_3 \geq 0$

Q2 Consider the following bipartite graph, where A, B, C and D are jobs and P, Q, R and S are officials contesting for these jobs. The number on the edge (i, j) denotes the profit earned by the company when the job 'i' is done by the employee 'j'. The CEO of the company wants to assign jobs so that no employee gets more than one job and no job is assigned to more than one employee so as to maximize the total profit.



Formulate an Integer Program to compute a maximum profit matching in the above graph. Write LP relaxation for this Integer Program. Consider the following fractional solution for above maximum weight matching problem: X_{AP}=0.3, X_{AS}=0.7, X_{BQ}=0.8, X_{BR}=0.2, X_{CP}=0.7, X_{CS}=0.3, X_{DQ}=0.2, X_{DR}=0.8. Apply cycle cancelling procedure to above solution to obtain an integral solution. Show all steps. Compare it with fractional solution. Can we obtain a non-integral optimal solution by solving above LP? Justify your answer.

Q3 Write the dual of the following problem.

Max $z = 5X_1 + 6X_2$

Subject to

$$X_1 + 2X_2 = 5$$

$$-X_1 + 5X_2 \geq 3$$

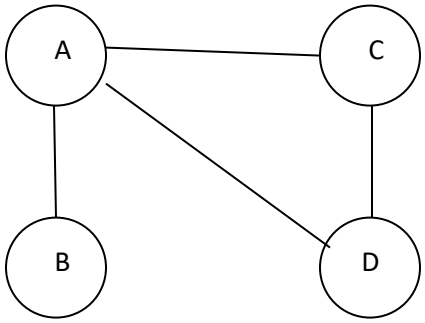
$$4X_1 + 7X_2 \leq 8$$

$$X_1 unrestricted\ , X_2 \geq 0$$

Solve the above problem using simplex method and find the solution of dual using the solution of primal problem

Verify the complementary slackness condition for the primal-dual pair.

Q4 Consider the graph given below:



Formulate an integer program to determine the Maximum independent set of the above graph. Also, formulate the LP Relaxation by relaxing the integrality constraint in above integer program. Solve the linear program to determine the optimal solution. Does the solution of linear program is the maximum independent set of the above graph. Justify.

Q5 Solve the following system of equations using two-phase method

Maximize z= $2x_1 + 2x_2 + 4x_3$

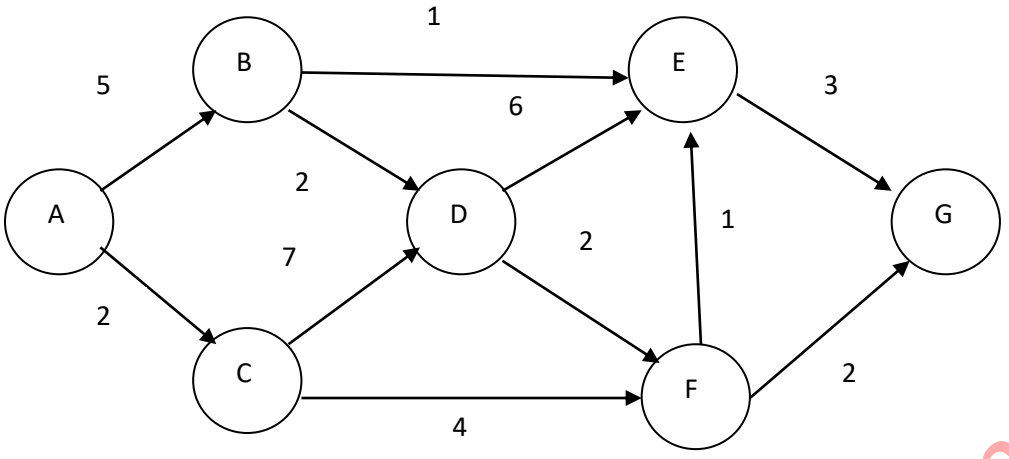
Subject to $2x_1 + x_2 + x_3 \leq 2$

$3x_1 + 4x_2 + 2x_3 \geq 8$

$x_1, x_2, x_3 \geq 0$

State how one can determine whether an LPP has a feasible solution? If a primal LPP has infeasible solution, what kind of solution would be possessed by its dual?

Q6 Determine the maximum flow from node A to node G for the following network using Ford Fulkerson method. The numbers represented on the edges are capacities of corresponding arcs. Show all steps of the calculation.



Let G be an arbitrary flow network, with source s, sink t, and positive "integer" capacity c(e) for every edge e. Consider a minimum s-t cut (S,T) of G. Construct G' by modifying the edge weight in G as follows: c'(e) = c(e)+1 (for each edge e). Now consider the cut (S,T) for G'. Is (S,T) a minimum cut for G' ? Justify your answer.